are destructive, some protective or constructive. Peat and coal are considered, so also Foraminifera, Radiolaria, and Corals. Part iii. is headed "Changes from Within," and the old Temple of Serapis comes before us as a witness to the fluctuations of level. Movements in the earth's crust, faults, flexures and overthrusts, and the formation of mountains are considered. We pass on to volcanic action and earthquakes, and eventually to the subjects of joints, spheroidal structures, nodules, and mineral veins, all of which are briefly noticed.

In Part iv. we come to to what some may consider as geology proper; or, "The Story of Past Ages." Naturally we most of us like to go back to the very beginning, and glean what knowledge we can of the earth's probable origin; and the author briefly reviews the evidence. More interesting, however, is the chapter which deals with the eras and subdivisions in geological history, and we feel at any rate on *terra firma*. In regard to questions of correlation, the views of Prof. Huxley on "homotaxis" are given more prominence than some students of "zones" will admit to be justified by the present state of our knowledge. Nevertheless, it must be admitted that the philosophy of zones has yet to be understood, and, as the author urges, "geological contemporaneity from the general similarity of fossils must not be pressed too far." Each area must be interpreted as far as possible by its own evidence, and when the sequence of organic remains is found to be alike in areas widely apart, even the highest authorities do not claim precise, but only approximate, contemporaneity.

The Archæan era is one to which the author has paid much attention. It is a complex and comprehensive group, and barren of life, so far as our present knowledge goes. Many crystalline schists and gneisses are provisionally referred to this division, and the author is inclined to regard their general characters as a witness to their age. He points out how evidence to the contrary has in several cases broken down. Still, an attitude of reserve is desirable, when we bear in mind that great earth-movements have taken place in Tertiary times, and that deep-seated metamorphism, as Mr. Barrow has shown in the south-eastern Highlands of Scotland, is all that is wanted to produce the results.

From the chaos of the Archæan era we pass on to a chapter dealing with "The Building of the British Islands," and we are furnished with a brief account of the Cambrian, Ordovician, and Silurian periods. Restorations of the possible geography of Britain in early Carboniferous and subsequent times are given, after Jukes-Browne, and we are led on stage by stage with brief descriptions through the series of formations. With regard to the origin of boulder clay, the agency of coast-ice is maintained in opposition to that of ice-sheets; but in this, as in other cases, where serious alternative opinions are held, the different views are stated.

"The Building of Europe and other Continents" is the next subject provided for us, and we are given a brief sketch of the history of the Alps. Then follows "A Sketch of the Earth's Life History," which might better have been incorporated with the accounts of the several formations whose history was previously told, for the leading types of life are treated stratigraphically. Here, in some respects, the popular meanings given to the names of fossils are more amusing than useful. Thus we have the *Microlestes* or "little thief," the *Dromatherium* or "running beast," the *Turritella* or "cockspur," the *Plesiosaurus* or "neighbour-lizard," the *Pliosaurus* "more nearly a lizard," and so on.

The concluding part is "On Some Theoretical Questions." It deals with the age of the earth, which "is long but it is very far from being boundless." The author, in fact, as stated in his preface, thinks that we can discover "by processes strictly inductive, some sign of its beginning and some foreshadowing of its end." We are not disposed to challenge the statement.

The permanence of ocean basins and of land areas is a well-worn theme, on which "agreement ceases"; but the author is far from dogmatic on the subject. The problems of climatal change and of the distribution of life on the earth, form the topics discussed in the closing chapters.

The general reader who peruses this volume cannot but gain a sound general knowledge of geological science. In the ordinary sense of the term we cannot call it a very "popular" book, for it contains too much solid reading to please those who would read for entertainment. It must be read by those desirous of instruction; by them it will be appreciated, and for them we believe the work has been written. To the more serious student it will be of service in regard to Prof. Bonney's views; but as a work of reference it lacks importance, from the fact that the author does not, as a rule, cite original authorities, but gives references mainly to text-books and other works of a general character. That he has laboured with enthusiasm, and lightened his labours with many a pleasant, and sometimes a pungent, remark on things in general, will be evident to all who take up this volume.

CAYLEY'S PAPERS.

The Collected Mathematical Papers of Arthur Cayley, Sc.D., F.R.S., Sadlerian Professor of Pure Mathematics in the University of Cambridge. (Cambridge: University Press, 1889, *et seq.*)

LATE in the year 1887 the Syndics of the University Press requested Prof. Cayley to allow his mathematical papers to be reprinted in a collected form. To this he acceded, and also undertook the work of superintending the impression and of adding such notes and references as appeared to him desirable.

Cayley's papers, commencing in the year 1841, have appeared in every periodical mathematical publication of importance in Europe and America. A worker in pure mathematics, in whatever special department of geometry or analysis, is well-nigh certain to find that his subject has come under Cayley's hand, so comprehensive are the researches that he has presented to the scientific world. A study of one or more of these papers being thus inevitable, it is easy to appreciate the importance to a student of this wonderful collection—wonderful alike in regard to extent, to variety, and to quality. It is not our intention to attempt a sufficient review of the six volumes that have appeared in the five years that have elapsed since January, 1889. If this were the end in view, a volume would be necessary. On this occasion we purpose to glance rapidly at some of the great discoveries, here set forth, which will be for ever linked to the Professor's fame.

The first volume comprises 100 papers, numbered consecutively and nearly in chronological order, produced between the dates 1841 and 1853. The paper No. 13, "On the Theory of Linear Transformations" (1845), marks a distinct epoch. Boole, in 1843, had proved the invariantive property of the discriminant of a quantic homogeneous in *m* variables, and Hesse, in 1844, had established certain covariantive properties of the ternary cubic. Here we find the general problem of "invariants" proposed, and some progress made towards its solution. The first step was the generalisation of Boole's theorem to a quantic of order *n*, containing *n* sets of *m* variables, the variables of each set entering linearly. This led to a function of the coefficients which possesses Boole's invariantive property, but it was not the discriminant as first pointed out by Ischläfli. This function, however, was seen by Cayley to necessarily satisfy a certain system of partial differential equations, and he was thence led to the capital discovery that a class of functions satisfied the same equations, and that each member of the class possessed the invariantive property. Other papers followed, and finally, in the year 1854, Cayley commenced the series of memoirs on Quantics which at intervals during the succeeding five-and-twenty years appeared in the *Philosophical Transactions* of the Royal Society. In this way the theory of algebraic invariants was gradually evolved. To this result there were many other contributors, notably Sylvester (to whom most of the nomenclature is due), Salmon, and Hammond in this country, and Aronhold, Clebsch, Gordan, and Hilbert in Germany. The first six memoirs are presented in vol. ii. of the collection. In vol. i. may be noted also the theory of conics of involution in connection with curves of the third order; the theory of "Pfaffiaus"; and the theory of surfaces of the third order. The last paper mentioned —a very important one—was developed in a correspondence with Dr. Salmon, to whom is attributed the enumeration of the twenty-seven lines on the surface. In vol. ii. we find in the notes which conclude the volume an account of the early bibliography of the theory of invariants; also the remarkable memoir on the theory of matrices, a subject which, at the present day, is exhibiting considerable vitality. The memoir appears to have been overlooked by mathematicians for more than twenty years after it appeared in 1858. The single exception appears to have been the paper by Laguerre, "Sur le Calcul des Systèmes Linéaires" (*Jour. Ec. Polyt.*, t. xxv.), in 1867. However, the subject was ultimately taken up by Sylvester, in his "Lectures on Multiple Algebra," in the *American Journal of Mathematics*, and is now an important branch of pure mathematics, both in its results and in its ideas. The volume is also remarkable for researches on the "Partitions of Numbers," "Skew Symmetric Determinants," the "Theory of Groups," "Caustics," and "Curves of the Third Order."

Vol. iii. contains notably the contributions to dynamics and astronomy. There is the valuable "Report on the recent Progress of Theoretical Dynamics," from the "Report of the British Association for the Advancement of Science, 1857." The review extends from Lagrange, 1788, to Bertrand, 1857. It, principally, gives an account of the notable contributions of Lagrange, Poisson, Sir W. R. Hamilton, and Jacobi. There are papers on Lunar Theory, Elliptic Motion, and the Problem of Three Bodies; also an extensive series of "Tables of the Development of Functions in the Theory of Elliptic Motion." The paper (No. 221) "On the Secular Acceleration of the Moon's Mean Motion" is interesting as supplying an independent verification of Prof. Adams' correction of Plana's expression for the true longitude.

Vol. iv. is chiefly remarkable for the "Report on the Progress of the Solution of certain Special Problems of Dynamics," from the Report of the British Association for the Advancement of Science for the year 1862. At the commencement the author adverts to a serious omission in his former report in volume iii. This has reference to the memoir by Ostrogradsky, "Mémoire sur les équations différentielles relatives au problème des Isopérimètres," *Mém. de St. Pét.* t. iv., 1850, which contains, in the most general form, the transformation of the equations of motion from the Lagrangian to the Hamiltonian form, and also the transformation of the system arising from any problem in the calculus of variations to the Hamiltonian form. The remark is also made that in a work by Cauchy, "Extrait du Mémoire présenté à l'Académie de Turin le 11 Oct., 1831," published in lithograph under date Turin 1832, there is satisfactory evidence that Cauchy, in the year 1831, was familiar with the Hamiltonian form of the equations of motion. Mention is made of papers on theoretical dynamics, notably by Bour, Jacobi, Natani, Clebsch, and Boole, which had appeared subsequent to the writing of the first report.

The second report includes various problems relating to the "particle" and the "solid body," "the problem of three bodies," &c. A list of memoirs is added, and increases the historical value of the report. One paper, No. 265, "Addition to the Memoir on an Extension of Arbogast's Method of Derivations," is published in this volume for the first time. The general subject of "quantics" is resumed in a seventh memoir, which is principally devoted to ternary forms. An important paper is that on the "Theory of Equations of the Fifth Order." The quintic equation cannot be solved algebraically. By the "resolution of the quintic" mathematicians understand the expression of its roots in terms of those of its resolvent sextic. During the fifteen years preceding 1861 considerable progress had been made with this problem by Cockle and Harley. In the year mentioned Cayley obtained a new auxiliary sextic equation, and showed that the roots of the quintic are, each of them, rational functions of its roots. A fresh impulse was given to the subject by George Paxton Young and Emory McClintock in vols. vi. and viii. of the *American Journal of Mathematics*, 1884-1886. The former indicated the general form of the roots of the quintic, and the latter made it possible to directly and visibly express them. In a long note at the end of the volume Prof. Cayley supplies the links between his own work and the later work of McClintock, which he considers very important and remarkable. In vol. v. there are two papers on the theory of curves in space, and in the "notes" the Professor gives his present views, taking into consideration the later work of Halphen, Nöther, Valentine, and Hilbert. These authors make considerable use of Cayley's conception of the monoid surface, the curve being regarded as the partial intersection of a cone with a monoid surface; Halphen obtained the fundamental theorem which shows the existence of a monoid surface of order $n + 1$, where n is the order of the cone of lowest order which passes through all the nodal lines of the aforesaid cone. Important remarks are made on the theory arising from this notion. In the Professor's view, the question of the *classification* of curves in space, according to the representation, as the partial intersection of a cone and a monoid surface is that which properly first presents itself.

There are many papers on geometry, including those on "skew surfaces" and "sextactic points of a plane curve," with which all geometricians are familiar.

The paper No. 347, "On the Notion and Boundaries of Algebra," considers in particular the line of separation between finite and transcendental analysis. The views of so eminent a man on this subject are necessarily of interest and importance. In stating algebra to be both an art and a science, the author is in agreement with the constantly reiterated opinion of Prof. Sylvester, and indeed with that of the great majority of those who have made algebra a subject of special study. The two great divisions of algebra are stated to be "tactic" and "logistic," and the remark is made that every algebraical theorem rests, ultimately, on a tactical foundation. Mathematicians will, we think, perceive in the word "tactic" a singularly felicitous expression to denote those operations which relate to arrangement of mat rial.

The paper No. 312, "On the Partitions of a Close," generalises Euler's polyhedral relation,

$$F + S = E + 2$$

and is a first step towards Listing's well-known developments.

The frontispiece of vol vi. is a portrait of Prof. Cayley, attired in gown, seated at his desk in the act of writing. The picture is somewhat dark and not quite so pleasing as those which of recent years have appeared in NATURE and in the *American Journal of Mathematics*. The volume is principally on geometry. The memoir No. 412, on "Cubic Surfaces," adopts a classification depending on the nature of the singularities. In the notes the notation is compared with that of Zeuthen, and several apparent discrepancies in the results are explained.

There is a long memoir on the polyzomal curves

$$\sqrt{U} + \sqrt{V} + \ldots. = 0$$

U, V, &c., being rational integral functions of the same degree in the variables. The general v-zomal curve (*i.e.* v functions U, V, &c.) is considered, and the branches, singularities, order, class, &c., determined. The investigation is intimately connected with Casey's work, on Bicircular Quartics, published in 1867 in the Proceedings of the Royal Irish Academy.

Other geometrical subjects considered are "Reciprocal Surfaces," "Skew and Developable Surfaces," "Abstract Geometry," "Cubical Divergent Parabolas," &c. In analysis we have the eighth memoir on quantics, which relates chiefly to the binary quintic; and, finally, mention may be made of the reproduction of Euler's memoir of 1758, on the rotation of a solid body.

The number of papers which appear in the six volumes is 416. Several hundreds of papers have yet to appear, and it seems improbable that a total of ten volumes will be found sufficient. This improbability is increased from the circumstance that Prof. Cayley is still producing a considerable amount of mathematical work.

For excellence of mathematical printing, and general care of production, unstinted approval may be awarded to the Cambridge University Press. These handsome volumes, as they appear, are rearing a fitting monument of the work of an eminent man, and are causing gratification and congratulation amongst mathematicians the world over.

P. A. MACMAHON.

THE PAMIRS.

The Pamirs: being a Narrative of a Year's Expedition on Horseback and on Foot through Kashmir, Western Tibet, Chinese Tartary, and Russian Cental Asia. By the Earl of Dunmore, F.R.G.S. In two volumes. (London: John Murray, 1893.)

LORD DUNMORE embodies in these volumes the journal of a somewhat remarkable journey of a year's duration, the initial and terminal points of which were Karachi and Constantinople. During most of the time he was accompanied by Major Roche, and the object of the journey was sport, especially the pursuit of the *Ovis poli*. Unfortunately this great sheep—of which Lord Dunmore (vol. ii. p. 56) appears to doubt the ovine character—inhabits a country the political geography of which is still undergoing spasmodic evolution, and there is reason to suspect the censorship of the Indian Government on all English writings bearing on the region. Thus we cannot expect any great contributions to the detailed topography of the Pamirs to be made public, and as the author makes no claim to any scientific acquirements, the botany and geology of the vast tracts wandered over are left no clearer than they were before. Major Roche, however, made an entomological collection. It is deeply to be deplored that every traveller, who is so fortunately circumstanced as to be able to push his way where few intelligent Europeans have been before, should not make some sort of preparation to fit himself for utilising his rare opportunities. For such preliminary instruction in science the Royal Geographical Society has made ample provision specially adapted to the wants of travellers, and as years go on we trust that the imputation of ignorance of what and how to observe may no longer be the necessary prelude to the criticism of works of travel in a scientific journal. Major Roche was fortunately provided with a camera, and made splendid use of it, although most of his negatives were unfortunately destroyed. Lord Dunmore himself is something of an artist, and much value must be attached to the landscapes which illustrate his book. Both travellers carried aneroids and, presumably, some surveying instruments, as a map with several new features is one of the results of the journey. We may point out that the larger-scale map of the Pamirs should have been